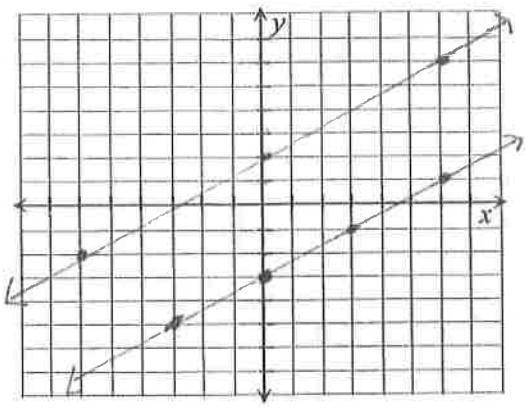
Name Key Class _____ Date _____

Solving Systems of Equations

Solve by graphing.

1. $y = \frac{2}{3}x - 3$
   $y = \frac{4}{6}x + 2$



No solution
(parallel lines)

Solve each system by substitution.

2. $-2x + y = -9$
   $3x - 12y = 24$

* isolate a variable first

$y = 2x - 9$

$3x - 12(2x - 9) = 24$
$3x - 24x + 108 = 24$
$-21x + 108 = 24$
$-21x = -84$
$x = 4$

$y = 2(4) - 9$
$y = -1$

(4, -1)

Solve each system by elimination.

4. $(-4x + 3y = 24)$ ×2
   $8x - 6y = -48$
   $-8x + 6y = 48$
   $0 = 0$

Infinite Solutions

Solve by any method.

6. $(4x - 5y = 11)$ ×7
   $(6x + 7y = 31)$ ×5

Set up for Elimination

$28x - 35y = 77$
$30x + 35y = 155$
$58x = 232$
$x = 4$

$6(4) + 7y = 31$
$24 + 7y = 31$
$7y = 7$

(4, 1)

3. $x + 3y = 9$
   $y = -\frac{3}{2}x + \frac{13}{2}$

$x + 3\left(-\frac{3}{2}x + \frac{13}{2}\right) = 9$
$x - \frac{9}{2}x + \frac{39}{2} = 9$
$-\frac{7}{2}x + \frac{39}{2} = 9$
$-\frac{2}{7} \cdot -\frac{7}{2}x = -\frac{21}{2} \cdot -\frac{2}{7}$
$x = 3$

$y = -\frac{3}{2}(3) + \frac{13}{2}$
$y = 2$

(3, 2)

5. $2(x + 2y = -7)$
   $3x - 4y = -1$
   $2x + 4y = -14$
   $5x = -15$
   $x = -3$

$-3 + 2y = -7$
$2y = -4$
$y = -2$

(-3, -2)

7. $x = 3y - 2$
   $x = 7y + 2$

Set up for Substitution

$3y - 2 = 7y + 2$
$-4y - 2 = 2$
$-4y = 4$
$y = -1$

$x = 3(-1) - 2$

(-5, -1)

Use a system of equations to solve each problem.

$P = 2l + 2w$ (perimeter)

8. Jamie has 300 ft of fencing to enclose a rectangular pasture. The pasture's length is to be 10 ft less than 3 times the width. Find the width of the garden.

$2l + 2w = 300$
$l = 3w - 10$

$2(3w-10) + 2w = 300$
$6w - 20 + 2w = 300$
$8w = 320$
$w = 40$

The width of the garden is 40ft.

9. You have a total of 25 coins, all nickels and quarters. The total value is \$3.85. Write and solve a system of equations to find the number of nickels n and the number of quarters q that you have.

$n + q = 25 \longrightarrow n = 25 - q$
$.05n + .25q = 3.85$
$.05(25-q) + .25q = 3.85$
$1.25 - .05q + .25q = 3.85$
$1.25 + .2q = 3.85$
$.2q = 2.6 \longrightarrow q = 13$

$n = 25 - 13$
$n = 12$

13 quarters and 12 nickels

10. You are planning to start a small business mowing your neighbors' lawns. You bought a used lawn mower for \$150 and you have calculated that an average lawn costs about \$1.75 in gasoline. You plan to charge \$20 per lawn.

a. Define your variables. y = amount of money
x = # of lawns

b. Write a cost function. $y = 1.75x + 150$

c. Write a revenue function. $y = 20x$

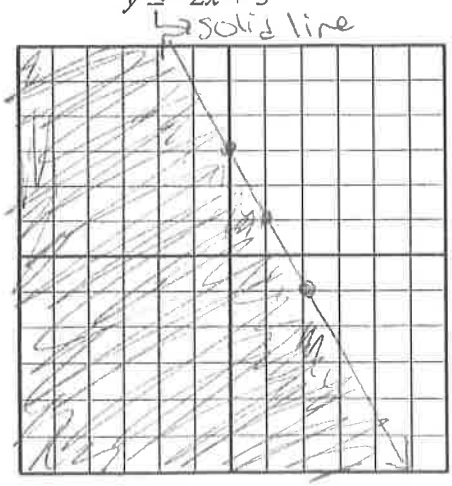
$20x = 1.75x + 150$
$18.25x = 150$
$x = 8.2\ldots$

d. Determine your breakeven point. Explain what the breakeven point means.
There is no breakeven point in this scenario (it would be ≈ 8.2 lawns)
this means at 8 lawns you would still owe money but at 9 lawns would start earning a profit.

Linear Inequalities/Systems of Linear Inequalities

Graph each inequality or system of inequalities.

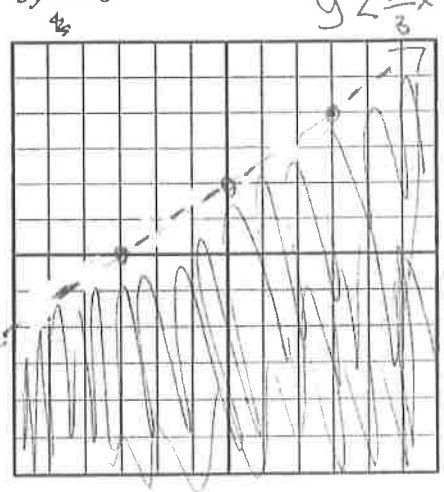
1. $y \leq -2x + 3$



2. $2x - 3y > -6$

$\frac{-3y}{-3} > \frac{-2x-6}{-3}$

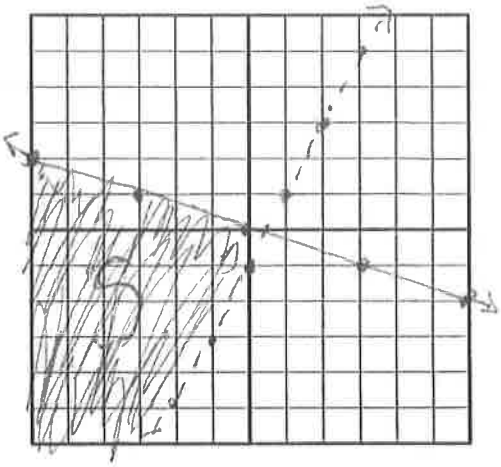
$y < \frac{2}{3}x + 2$



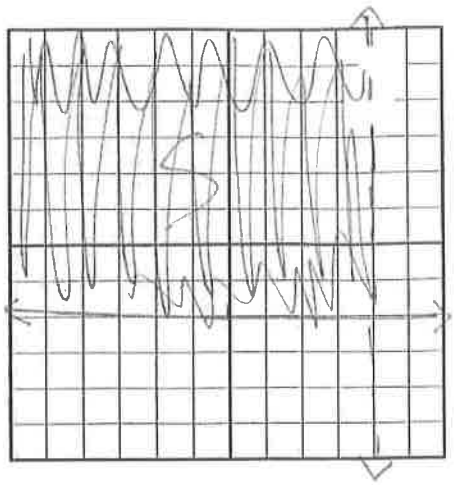
*flip inequality when x or ÷ by a negative #

3. $2x - y < 1$ $\quad -y < -2x + 1$ $\quad y > 2x - 1$

$y \leq -\frac{1}{3}x$



4. $x < 4$
$y \geq -2$



Exponential Functions

Simplify each expression.

1. $3x^5y \cdot 5x^2y^3$

$15x^7y^4$

2. $\left(\frac{y^5}{x^4}\right)^{-3} = \left(\frac{x^4}{y^5}\right)^3 = \frac{x^{12}}{y^{15}}$

3. $a^1b^0(a^{-5})$

$a^1 \cdot a^{-5} = a^{-1} = \frac{1}{a}$

4. $\frac{2y^4}{x^2y^{-4}} = \frac{2y^4 \cdot y^4}{x^2} = \frac{2y^8}{x^2}$

5. $(3xy^4)^{-4} = \frac{1}{(3xy^4)^4} = \frac{1}{81x^4y^{16}}$

6. $\frac{a^2b^8}{(2a^4b^2 \cdot a^2b)^2} = \frac{a^2b^8}{(2a^6b^3)^2}$

$\frac{a^2b^8}{4a^{12}b^6} = \frac{b^2}{4a^{10}}$

7. $\frac{(-2y)^4 \cdot x^4y}{-y^{-1}} = \frac{16y^4 \cdot x^4y}{-y^{-1}}$

$\frac{16x^4y^5}{-y^{-1}} = \frac{16x^4y^5 \cdot y^1}{-1} = -16x^4y^6$

8. $\frac{a^0b^3}{(2a^{-1}b^3)^{-1} \cdot a^{-2}} = b^3 \cdot (2a^{-1}b^3)^1 \cdot a^2$

$2ab^6$

9. $\frac{-5x^{-3}y^0 \cdot -2x^{-1}y^{-5}}{(4x^2y^{-4})^{-2}}$

$\frac{10x^{-4}y^{-5}}{(4x^2y^{-4})^{-2}} = \frac{10 \cdot (4x^2y^{-4})^2}{x^4y^5}$

$= \frac{10 \cdot 16x^4y^{-8}}{x^4y^5}$

$= \frac{160}{y^{13}}$

10. **Rewrite the following expressions using roots:**

a) $4x^{\frac{2}{3}}$ (2 = power, 3 = root) $\quad 4\sqrt[3]{x^2}$

b) $(8x)^{\frac{5}{2}}$ (5 = power, 2 = root) $\quad \sqrt{(8x)^5}$ or $\left(\sqrt{8x}\right)^5$

11. **Rewrite the following expressions using fractional exponents:**

7) $\sqrt[5]{x^1}$ $\quad x^{\frac{1}{5}}$

8) $\left(\sqrt{4x}\right)^3$ $\quad (4x)^{\frac{3}{2}}$

12) Which function, $f(x)=2(3.2)^x$ *or* $f(x)=2(3.3)^x$ is steeper? Explain how you know.

$f(x)=2(3.3)^x$ is steeper because when $b>1$ the larger the base (b) the steeper the curve.

13) Which function, $f(x)=2\left(\frac{5}{4}\right)^x$ *or* $f(x)=2\left(\frac{4}{5}\right)^x$ is decreasing? Explain how you know.

$f(x)=2\left(\frac{4}{5}\right)^x$ is decreasing because the base is in between 0 and 1 ($0<b<1$).

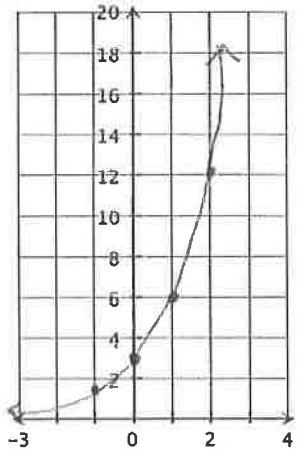
14) In terms of exponents, what is the meaning of $8^{\frac{1}{3}}$?

This means what number times itself 3 times (or what number to the 3rd power) equals 8. $2^3=8$ so $8^{\frac{1}{3}}=2$.

15) Graph $y=3(2)^x$

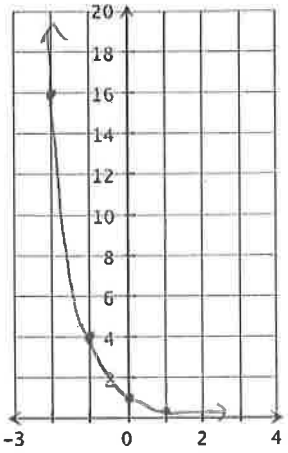
multiplier

y-intercept



16) Graph $y=\frac{1^x}{4}=1\left(\frac{1}{4}\right)^x$

y-int = 1



17)

x	0	1	2	3	4	5
y	36	12	4	$\frac{4}{3}$	$\frac{4}{9}$	$\frac{4}{27}$

a) Does this data represent an exponential function? Explain.

Yes, because there is a constant multiplier of $\frac{1}{3}$.

b) What is the equation for this function?

$y=36\left(\frac{1}{3}\right)^x$

18)

x	0	1	2	3	4	5
y	4	12	20	28	36	44

a) Does this data represent an exponential function? Explain.

No, the data represents a linear function because there is a constant rate of change of 8.

b) What is the equation for this function?

$y=8x+4$

$y = ab^x$ (a) $b = 1 + .15$

19) The starting salary for a new employee is $35,000. The salary for this employee increases by 15% per year. What is the salary after 15 years? $y = 35{,}000\,(1.15)^{15}$

$\$284{,}797.16$

20) You drink a soda with 120mg of caffeine. Each hour the caffeine in your system decrease by about 12% each hour. Approximately how much caffeine is in your system after 8 hours? $b = 1 - .12 = .88$

$y = 120\,(.88)^8$

43.2 mg

21) Gold-198 has a half-life of 2.7 days. How much of a 96 gram sample of gold-198 will be left after 8.1 days?

halves once every 2.7 days (a) # of half lives $= \frac{8.1}{2.7} = 3$

$y = 96\left(\frac{1}{2}\right)^3 =$ 12g

22) You just joined Twitter. Your initial number of Twitter followers is 2. You find that your number of followers triple every 2 weeks. How many followers will you have after 10 weeks? $b = 3$ (a)

# of tripling periods: $\frac{10}{2} = 5$

$y = 2(3)^5$ 486 followers

23) Suppose you have saved up \$4,000 in an account that pays 2% interest compounded semi-annually. What is your account balance after 20 years? $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $n = 2$ t 20

$A = 4{,}000\left(1 + \frac{.02}{2}\right)^{2 \cdot 20}$

$A = 4{,}000\,(1.01)^{40} =$ \$5,955.45

Simplifying/Factoring Polynomials.

Name the polynomial using the degree and number of terms:

1. $-3x^3 + 5x^2 + 2$

Cubic trinomial

2. $-2a^4b^3 + a^3b^2c - 6a^2bc + 7abc$

7th degree polynomial of 4 terms.

3. $8x - 4$

linear binomial

4. 7

constant monomial

Simplify each expression.

5. $(3x^2 + x) + 7 + 7x + 8x^4 - 3$

$8x^4 + 3x^2 + 8x + 4$

6. $(8y^2 - 4y + 5) - (y^2 + 5y - 3)$ *distribute -1 first

$8y^2 - 4y + 5 - y^2 - 5y + 3$

$7y^2 - 9y + 8$

7. $2x(7x - 3) - 4x$

$14x^2 - 6x - 4x$

$14x^2 - 10x$

8. $3x(2x + x^2) + 5x(3x - 4x^2)$

$6x^2 + 3x^3 + 15x^2 - 20x^3$

$-17x^3 + 21x^2$

9. $(x+7y)(x-4y)$

$x^2-4xy+7xy-28y^2$

$\boxed{x^2+3xy-28y^2}$

10. $(2x^2-3)(8x^2+x)$

$\boxed{16x^4+2x^3-24x^2-3x}$

11. $(w^2+3)(3w^2+5w-6)$

$3w^4+5w^3-6w^2+9w^2+15w-18$

$\boxed{3w^4+5w^3+3w^2+15w-18}$

12. $(3x+5)(2x-7)$

$6x^2-21x+10x-35$

$\boxed{6x^2-11x-35}$

Simplify each expression.

13. $(w-5)^2$ $(w-5)(w-5)$

$\boxed{w^2-10w+25}$

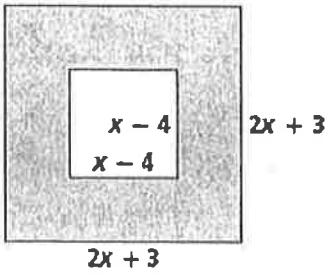
14. $(3x+8)^2$ $(3x+8)(3x+8)$

$\boxed{9x^2+48x+64}$

15. $(x+13)(x-13)$

$\boxed{x^2-169}$

16. Find the area of the shaded region. $A_{whole} - A_{unshaded}$



$A_W = (2x+3)(2x+3)$

$A_W = 4x^2+12x+9$

$A_U = (x-4)(x-4)$

$A_U = x^2-8x+16$

$(4x^2+12x+9) - (x^2-8x+16)$

$4x^2+12x+9-x^2+8x-16$

$\boxed{3x^2+20x-7}$

Factor each expression.

17. $49x^4-16y^2$ Difference of squares

$\boxed{(7x^2-4y)(7x^2+4y)}$

18. $6x^4y^3+18x^2y^2-24xy$ GCF only

$\boxed{6xy(x^3y^2+3xy-4)}$

19. $x^2-8x-48$

$\boxed{(x-12)(x+4)}$

20. $2x^2+15x+13$

$\boxed{(2x+13)(x+1)}$

21. $3x^2-48x+189$ *GCF + trinomial $a=1$ factoring

$3(x^2-16x+63)$

$\boxed{3(x-9)(x-7)}$

22. $4x^2-12xy+9y^2$ Perfect square trinomial

$\boxed{(2x-3y)^2}$

23. $24x^2-92x+28$ $a\cdot c = 42$, $-21 \cdot -2$

$4(6x^2-23x+7)$

$4(6x^2-2x-21x+7)$

$4(2x(3x-1)-7(3x-1))$

$\boxed{4(3x-1)(2x-7)}$

24. $x^2+9xy-36y^2$

$\boxed{(x+12y)(x-3y)}$

25. $64y^2-100$ Difference of squares with a GCF

$4(16y^2-25)$

$\boxed{4(4y-5)(4y+5)}$

26. $4n^3+8n^2-5n-10$ Grouping

$4n^2(n+2)-5(n+2)$

$\boxed{(n+2)(4n^2-5)}$

Graphing/Solving Quadratics.

1. Without graphing, describe the features of the graph of $y = -5x^2 + 2x + 7$

The parabola opens down and has a maximum because $a<0$. The parabola is narrow because $|a|>1$. The y-intercept is $(0,7)$.

2. Without graphing, describe the features of the graph of $y = \frac{1}{2}(x+3)^2 - 4$

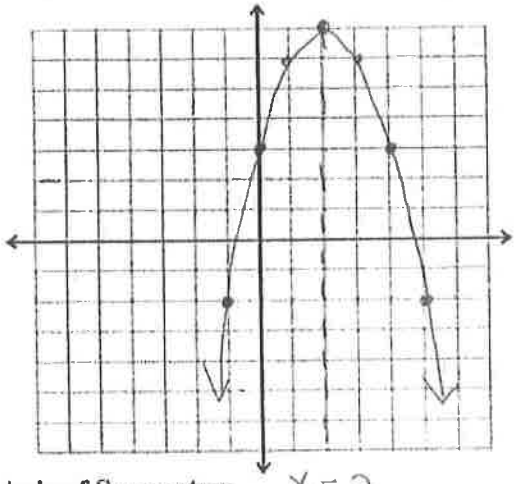
The parabola opens up and has a minimum because $a>0$. The parabola is wide because $0<|a|<1$. The vertex is $(-3,-4)$.

Find the missing information and graph each quadratic. You should have 5 points plotted and graph the axis of symmetry

3. $y = -x^2 + 4x + 3$

$x = \frac{-4}{2(-1)} = \frac{-4}{-2}$

$x = 2$



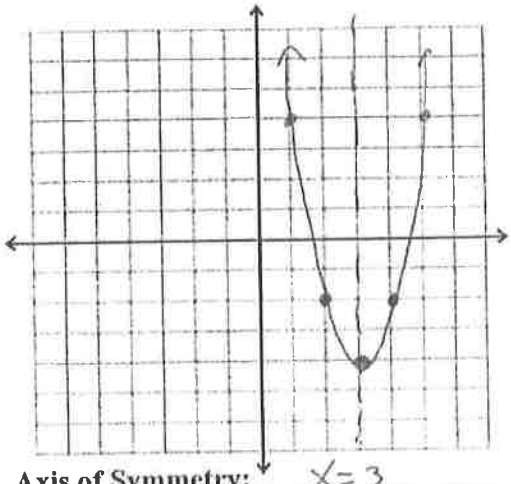
Axis of Symmetry: $x = 2$

Vertex: $(2, 7)$

Y-Intercept: $(0, 3)$

Equation in Vertex Form: $y = -(x-2)^2 + 7$

4. $y = 2(x-3)^2 - 4$



Axis of Symmetry: $x = 3$

Vertex: $(3, -4)$

Y-Intercept: $(0, 14)$

*Plug in 0 for x

$y = 2(0-3)^2 - 4$

$y = 2(-3)^2 - 4$

$y = 2(9) - 4$

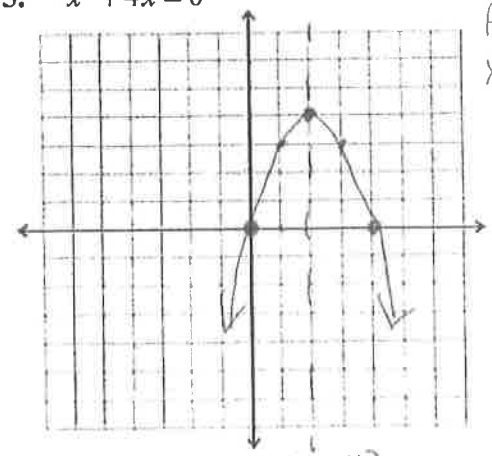
$y = 14$

Solve by graphing:

5. $-x^2 + 4x = 0$

$a=-1$, $b=4$, $c=0$

A.O.S.: $x = \frac{-4}{2(-1)} = 2$



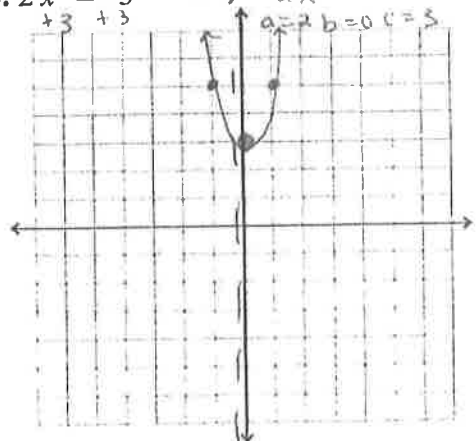
Solution (s): $x = \{0, 4\}$

6. $2x^2 = -3$ → $2x^2 + 3 = 0$

$+3 \quad +3$

$a=2$ $b=0$ $c=3$

A.O.S. $x = 0$



Solution (s): No real solutions

7. In the game Angry Birds the yellow angry bird is flung through a sling shot and follows a parabolic path. The flight of the path of the bird can be modeled with the equation $y=-16(x-1)^2+38$. What is the maximum height the yellow angry bird will reach?

vertex (1, 38) — vertex
time, height

38 ft

8. A ball is thrown into the air with an initial upward velocity of 60 ft/s. Its height h in feet after t seconds is given by the function $h=-16t^2+60t+6$.

a. What is the maximum height of the ball?

vertex
$t=\frac{-60}{2(-16)}$
$t=1.875$ seconds

$h=-16(1.875)^2+60(1.875)+6$

$h=62.25$ ft

b. After how many seconds will the ball hit the ground?

$h=0$ $0=-16t^2+60t+6$

$t=\frac{-60\pm\sqrt{60^2-4(-16)(6)}}{2(-16)}=\frac{-60\pm\sqrt{3984}}{-32}$

$-.097$ 3.85

3.85 seconds

9. The profits of a company can be modeled by the equation $P=300x^2-1000x-100$ where x is the number of years and P is the amount of profit in thousands of dollars. How long will it take the company to break even? Round your answer to the nearest tenth.

$P=0$

$0=300x^2-1000x-100$

$x=\frac{1000\pm\sqrt{(-1000)^2-4(300)(-100)}}{2(300)}=\frac{1000\pm\sqrt{1120000}}{600}$

3.43 years $-.10$

Solve by taking the square root. Leave your answers in simplest radical form.

10. $2m^2-1=97$

$2m^2=98$

$m^2=49$

$m=\pm7$

11. $3g^2+81=0$

$3g^2=-81$

$g^2=-27$

No real solutions

12. $16x^2=1$

$x^2=\frac{1}{16}$

$x=\pm\sqrt{\frac{1}{16}}$

$x=\pm\frac{1}{4}$

13. $2x^2+18=118$

$2x^2=100$

$x^2=50$

$x=\pm\sqrt{50}$

$\pm\sqrt{25}\sqrt{2}$

$x=\pm5\sqrt{2}$

Solve by factoring.

14. $m^2+m-56=0$

$(m+8)(m-7)=0$

$m+8=0$ $m-7=0$

$m=-8$ $m=7$

$m=\{-8,7\}$

15. $c^2=8c$

$c^2-8c=0$

$c(c-8)=0$

$c=0$ $c-8=0$

$c=8$

$c=\{0,8\}$

16. $4x^2-11x=-6$

$a\cdot c=24$

$-8\cdot-3$

$4x^2-11x+6=0$

$4x^2-8x-3x+6=0$

$4x(x-2)-3(x-2)=0$

$(x-2)(4x-3)=0$

$x-2=0$ $4x-3=0$

$x=2$ $x=\frac{3}{4}$

$x=\{\frac{3}{4},2\}$

17. $5x^2=-40x-35$

$5x^2+40x+35=0$

$5(x^2+8x+7)=0$

$5(x+7)(x+1)=0$

$x+7=0$ $x+1=0$

$x=-7$ $x=-1$

$x=\{-7,-1\}$

Use the quadratic formula to solve each equation. Leave answers in simplified radical form.

14. $x^2 - 4x - 7 = 0$

$x = \frac{4 \pm \sqrt{16-4(1)(-7)}}{2(1)}$

$x = \frac{4 \pm \sqrt{44}}{2}$

$\sqrt{44}$ → $\sqrt{4} \cdot \sqrt{11}$ → $2\sqrt{11}$

$x = \frac{2^{4} \pm {}^{1}2\sqrt{11}}{2_{1}}$

$x = 2 \pm \sqrt{11}$

15. $2x^2 - 5x - 12 = 0$

$x = \frac{5 \pm \sqrt{25-4(2)(-12)}}{2(2)}$

$x = \frac{5 \pm \sqrt{121}}{4}$

$\frac{5+11}{4} = 4$

$\frac{5-11}{4} = -1.5$

$x = \{-1.5, 4\}$

Use any method you choose to solve each equation.

16. $x^2 = -6x - 5$ factoring

$x^2 + 6x + 5 = 0$

$(x+5)(x+1) = 0$

$x+5 = 0$ $\quad x+1 = 0$

$x = -5$ $\quad x = -1$

$x = \{-5, -1\}$

17. $3x^2 - 12x = -1$ Quadratic formula

$3x^2 - 12x + 1 = 0$

$x = \frac{12 \pm \sqrt{144-4(3)(1)}}{2(3)} = \frac{12 \pm \sqrt{132}}{6}$

$\sqrt{132}$ → $\sqrt{4} \cdot \sqrt{33}$ → $2\sqrt{33}$

$= \frac{{}^{6}12 \pm {}^{1}2\sqrt{33}}{6_{3}}$

$= \frac{6 \pm \sqrt{33}}{3}$

18. $8k^2 + 3 = 139$ Square-root (no x term)

$8k^2 = 136$

$k^2 = 17$

$k = \pm\sqrt{17}$

19. $15x^2 - 20x + 5 = 0$ factoring

$5(3x^2 - 4x + 1) = 0$

$5(3x^2 - 3x - x + 1) = 0$

$5(3x(x-1) - 1(x-1)) = 0$

$5(3x-1)(x-1) = 0$

$3x - 1 = 0$ $\quad x - 1 = 0$

$x = \frac{1}{3}$ $\quad x = 1$

$x = \{\frac{1}{3}, 1\}$

Find the number of solutions of each equation. Explain how you arrived at your answer.

Discriminant!

20. $5x^2 - 4x + 6 = 0$

$b^2 - 4ac$

$(-4)^2 - 4(5)(6)$

-104

NO real solution because the discriminant is negative.

21. $3a^2 - 4a - 5 = 0$

$(-4)^2 - 4(3)(-5)$

76

2 solutions because the discriminant is positive

22. $16x^2 + 56x + 49$

$(56)^2 - 4(16)(49)$

0

1 solution because the discriminant is 0.