Topic: FACTORING (math skill for a lifetime of math!) without the calculator AGAIN!

Types of factoring: GCF

- Monomial
- Non-binomial

Binomials

- Difference of squares
- Sum of cubes
- Difference of cubes

Trinomials (leading coefficient 1, leading coefficient not 1)

- Trial and error (guess and check)
- Split the middle term
- Using the calculator (for honors students with weak math skills which is a contradiction!)
- Recognizing perfect square trinomials

Four or more terms

- Four terms- group by twos
- Four terms- group three by one or one by three
- More than four- look for patterns

Complete factoring

Prime factors

The best strategy for factoring a polynomial:

1. If there is a common factor, factor the GCF out **first.**
2. Determine the number of terms in the polynomial and try factoring as follows.
   a. If there are two terms, can the binomial be factored by one of the following special forms?
      Difference of two squares: $A^2 - B^2 = (A + B)(A - B)$
      Sum of two cubes: $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$
      Difference of two cubes: $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
   b. If there are three terms, is the trinomial a perfect square trinomial? If so factor by one of the following special forms:
      Binomial sum squared: $A^2 + 2AB + B^2 = (A + B)^2$
      Binomial difference squared: $A^2 - 2AB + B^2 = (A - B)^2$
      If the trinomial is not a perfect square trinomial, try factoring by trial and error or grouping (split the middle term).
   c. If there are four or more terms, try factoring by grouping.
3. Check to see if any factors with more than one term in the factored polynomial can be factored further. If so, **factor completely.**
4. Check by multiplying, if time permits.

Factoring, a little of everything:

GCF

1. $30x - 45$

2. $12x^3 + 16x^2 - 400x$

3. $30x^4y + 15x^3y^2 + 5x^2y$

4. $7(x+3) - m(x+3)$

5. $a(x-y) + b(x-y) - 2(x-y)$

Difference of squares

6. $m^2 - 9$

7. $4y^2 - 1$

8. $81 - 121y^2$

9. $25a^2 - 49b^2$

10. $w^4 - 16$

11. $16x^4 - 81y^4$

Trinomials, leading coeff. of 1

12. $m^2 - 3m + 2$

13. $x^2 - x - 20$

14. $x^2 + 19x + 48$

15. $x^2 - 6xy + 8y^2$

16. $y^2 + 5y - 9$

17. $3w^2 + 6w - 24$

18. $3m^3 - 36m^2 + 33m$

Perfect square trinomials

19. $x^2 + 22x + 121$

20. $k^2 - 16k + 64$

21. $9y^2 - 30y + 25$

22. $25x^6 - 30x^3 + 9$

23. $36d^2 + 60df + 25f^2$

24. $25x^2 - 40xy + 16y^2$

25. $100 - 20g + g^2$

Trinomials, leading coeff. not 1!

26. $3y^2 + 17y + 10$

27. $5m^2 - 17m + 6$

28. $4x^2 + 4x - 15$

29. $5y^2 + 11y + 4$

30. $8x^2 + 8x - 6$

31. $2x^2 + 11x + 5$

32. $15y^2 + 13y - 2$

33. $3x^2 - 5x + 1$

34. $16y^2 - 46y + 15$

35. $8x^2 - 22x + 5$

36. $9y^2 + 5y - 4$

37. $15w^2 - 19w + 6$

37. $3x^2 + 4xy + y^2$

38. $6x^2 - 7xy - 5y^2$

39. $12x^2 - 25x + 12$

40. $15m^2 - 31m + 10$

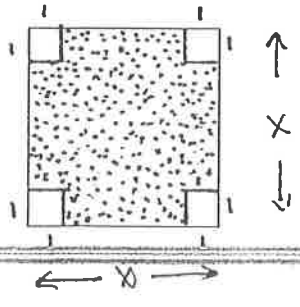
41. $12w^2 - 33w + 21$

42. $3x^3 + 14x^2 + 8x$

43. $24x^2 + 23x - 12$

44. $50m^2 + 35m + 6$

45. Find a formula for the area of the shaded region and express it in factored form.



Factoring by grouping by two's:

1. $x^3 + 4x^2 + x + 4$

2. $3y^3 - y^2 + 6y - 2$

3. $2x^3 + x^2 - 6x - 3$

4. $ax + bx - ay - by$

5. $a^2b^2 - 9a^2 - 4b^2 + 36$

6. $-9y^3 - 3y^2 + 3y + 1$

7. $x + 1 + x^3 + x^2$

8. $x^6 - 4x^4 - x^2 + 4$

Factoring that involves terms that are binomials!

9. $(3x + 2)^2 + 8(3x + 2) + 12$

10. $(a+b)^2 - 9$

11. $49b^2 - (a - b)^2$

Name: _____

Four term polynomials.

1. $x^2 + 10x + 25 - y^2$

2. $4m^2 + 20m + 25 - k^2$

3. $w^2 - 10w + 25 - 100z^2$

4. $49y^2 - 14y + 1 - 36x^2$

5. $k^2 + 18k + 81 - 25m^2$

6. $16x^2 - 56x + 49 - 64y^2$

7. $x^2 - 2xy + y^2 - 1$

8. $x^4 + 16x^2 + 64 - 121p^2$

9. $y^2 - x^2 - 10x - 25$

10. $k^2 - 4m^2 - 20m - 25$

11. $100z^2 - w^2 + 10w - 25$

12. $36x^2 - 49y^2 + 14y - 1$

13. $25m^2 - k^2 - 18k - 81$

14. $64y^2 - 16x^2 + 56x - 49$

15. $m^2 - x^2 + 2xy - y^2$

16. $121p^2 - x^4 - 16x^2 - 64$

Name: _____

Honors Integrated Math 3
More Practice Factoring

Mixed practice:

1. $-2x^3 + 16x^2$
2. $-2x^3 + 32x$
3. $(z + x)^2 - 5(z + x)$
4. $6 + y - y^2$
5. $8m^2 - 14m - 15$
6. $5w^2 + 25w - 120$
7. $49a^2 - 100b^2$
8. $100x^4 + 140x^2 + 49$
9. $12k^3 + 18k$
10. $2x^2 + 5x + 3$
11. $9y^2 - 36y - 45$
12. $6p^2 - 5pq - 6q^2$
13. $8x^6 + 10x^3 + 3$
14. $25d^2 - 10df + f^2$
15. $28x^{12}y^{10} - 16x^8y^6 + 4x^5y^3$
16. $6m^2 + m - 12$
17. $4y^2 - 13yz - 12z^2$
18. $625 - m^4$
19. $x^4 - 2x^2 + 1$
20. $a^2y - b^2y$
21. $(x+1)^2 - 2(x+1) + 1$
22. $144y^2 - 96y + 16$

More practice . . .

$3x^2(x-7) + 2x(x-7) - 5(x-7)$

$4a^2 - (a-2b)^2$

$x^2 - 4x + 4 - 9y^2 + 6y - 1$

$m^4 - 13m^2 + 36$

$x^4 - (2x-1)^2$

$w^4 - 13w^2 + 36$

$8(2p+q)^2 - 10(2p+q) + 3$

$x^2 - xz - xy + yz$

Sum and Difference of Cubes

$125x^3 - 1$

$8y^3 + 27$

$64m^3 + 27n^3$

$1000 - 343w^3$

$x^3 + a^3$

$8 + (a+ b)^3$

$(x+1)^3 - (y+2)^3$

$x^3 + y^3 + x^2 - y^2$

Name: _____

Honors Integrated Math 3
Factoring Practice Mastery

Get good at it. You won't regret it!

TYPE 1 : COMMON FACTORS

1. $x^2 + x$
2. $2m - 10$
3. $5x^3 - 50ax$
4. $12r^5 + 18r^3 x^2$
5. $13m^3n - 26m^2n^2 - 65mn^3$

TYPE 2: FACTORING BY GROUPING

1. $2x^3 - 3x^2 - 4x + 6$
2. $10y^3 - 15y^2 + 2y - 3$
3. $8m^2 - 12am + 6mn - 9an$
4. $12a^2x - 4a^2 - 9bx + 3b$
5. $24x^3 - 18x^2 + 60x - 45$

TYPE 3: QUADRATIC TRINOMIALS

1. $x^2 + 5x - 14$
2. $3m^2 + 9m - 30$
3. $8x^2 + 6xy + y^2$
4. $3r^2 + rs - 10s^2$
5. $6k^2 + 11k + 3$
6. $10x^2 - 29x + 21$
7. $10y^2 - 29y - 21$
8. $a(a+1)x^2 + x - a(a-1)$
9. $(2a+b)y^2 - (a-b)y - (a+2b)$
10. $12x^2 + 29x - 10$

TYPE 4: DIFFERENCE OF TWO SQUARES

1. $m^2 - 16$
2. $4x^2 - 9$
3. $y^2 - 1$
4. $36x^2 - 49y^2$

More difference of squares

5. $5x^2 - 80$
6. $3k^2 - 27$
7. $(x+7)^2 - 9$
8. $x^5y^3 - x^3y^5$
9. $36 - (x-5)^2$

TYPE 5: PERFECT SQUARE TRINOMIALS

1. $x^2 - 4x + 4$
2. $25m^2 + 10m + 1$
3. $49y^2 - 28y + 4$
4. $100x^2 - 60xy + 9y^2$

HARD TO FACTOR TRINOMIALS

1. $12m^2 + 25m + 12$
2. $24x^2 - 121x + 5$
3. $18w^2 + 27w + 10$
4. $30f^2 + 41f - 6$
5. $36x^2 - 63x + 20$
6. $35x^2 - 2xy - 6y^2$
7. $12x^2 - 179x - 15$
8. $200x^2 + 510x - 1001$

MIXED PROBLEMS

1. $a^2b^4 - a^4b^2$
2. $7a^2x - 6a^2 - 7x + 6$
3. $30x^2 + 95x + 50$
4. $x^4 - 26x^2 + 25$
5. $7x^4 - x^2 - 12$
6. $100 - (x - y)^2$

More mixed practice

7. $(2x + 3y + a)^2 - (x - y + a)^2$

8. $343 - 7(x+3)^2$

9. $x^2 - 13x - 30$

10. $y^2 - 6y + 7$

Factoring disguised difference of squares

1. $x^2 + 6x + 9 - y^2$

2. $a^2 - b^2 + 2b - 1$

3. $p^2 - 14p + 49 - 9k^2$

4. $r^2 - s^2 - 4s - 4$

5. $81 - x^2 + 2xy - y^2$

Factoring by completing the square!

Example: $x^4 + 2x^2 + 9 = x^4 + 6x^2 + 9 + 2x^2 - 6x^2 = x^4 + 6x^2 + 9 - 4x^2 = (x^2 + 3)^2 - 4x^2$

$= ((x^2 + 3) - 2x) ((x^2 + 3) + 2x) = (x^2 - 2x + 3)(x^2 + 2x + 3)$

1. $x^4 - x^2 + 16$

2. $x^4 + 11x^2 + 36$

3. $x^4 - 19x^2 + 9$

4. $x^4 + 4$

5. $x^4 - 26x^2 + 25$

Name: _____ More factoring practice: Notes

Pattern Factoring:

Difference of Squares:	$a^2 - b^2 = (a + b)(a - b)$
Perfect Square Trinomial:	$a^2 + 2ab + b^2 = (a + b)(a + b) = (a + b)^2$
	$a^2 - 2ab + b^2 = (a - b)(a - b) = (a - b)^2$
Difference of Cubes:	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
Sum of Cubes:	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

***Always factor out the greatest common factor first
***If two terms, check for difference of squares, difference of cubes or sum of cubes
***If three terms, factor as trinomial into two binomial factors.
*** If four terms, factor by grouping by pairing two or three terms together.

Practice factoring the following:

$a^3 + b^3 = (\quad)(\quad)$ $8x^3 + 27 = (\quad)(\quad)$

$1000x^3 + 1 = (\quad)(\quad)$ $125m^6 + 64 = (\quad)(\quad)$

$y^3 + 1 = (\quad)(\quad)$ $a^3 + 8 = (\quad)(\quad)$

$27m^9 + 8 = (\quad)(\quad)$ $(a + b)^3 + 216$

Challenge: $x^6 - y^6$ $m^9 - 1$

Mixed practice:

$49m^2 + 70m + 25$	$100x^3y^2 - 324xy^4$	$2(m+1)^2 - (m+1) - 15$	$18f^2 - 27f + 10$
$m^2 + 6m + 9 - n^2$	$4m^2 - (6m - 7)^2$	$a(a - b)^2 - a + b$	$a^2 + 2a + 1 - b^2 + 2b - 1$
$a^2 - b^2 + 8bc - 16c^2$	$8k^2 + 38k + 45$	$343 + 8w^3$	$(a^2 + 3a + 2) - x(a^2 + 3a + 2)$
$x^3 + x^2 - x - 1$	$(2x + 3y) + x(2x + 3y)$	$64a^6 - 1$	$x^3(x^2 + 4x + 4) - (x^2 + 4x + 4)$

Algebra 2: Name: _____

Use this set of problems to review Alg. II.

The following units are covered on the exam.

Unit Number and Topic	Sections covered	Pages for Chapter Review and Test	Pages for Extra skills and Word Problems
Unit 6: Polynomials	6-1 through 6-6, 6-8	Pages 359-362	Pages 846-847
Unit 7: Radicals & Rational Exponents	7-1 through 7-8	Pages 423- 426	Pages 848-849
Unit 8:Exponential & Logarithmic Functions	8-1 through 8-6	Pages 479-482	Pages 850-851
Unit 9: Rational Functions	9-1 through 9-6	Pages 539-542	Pages 852-853

The table above provides you with detailed information about where to look in your textbook for review purposes. The chapter reviews provide a summary of the key concepts of the unit, and the chapter test provides assorted examples of test practice problems in addition to this practice packet. The "two-page" Extra Practice in the back of the book provides additional problems. Use these resources, along with your notes, and your assessments to prepare for your Algebra 2 Final Exam.

POLYNOMIAL OPERATIONS

1. $(2x^2 + 3x + 5) + (4x^2 - 10x - 2)$

2. $(2x - 5)^2$

3. $(3x + 5)(9x^2 - 15x + 25)$

4. $(6x - 5y)(6x + 5y)$

. $xy^3(xy^2 + xy - 1)$

POLYNOMIAL FACTORING

6. $x^3 - 27$

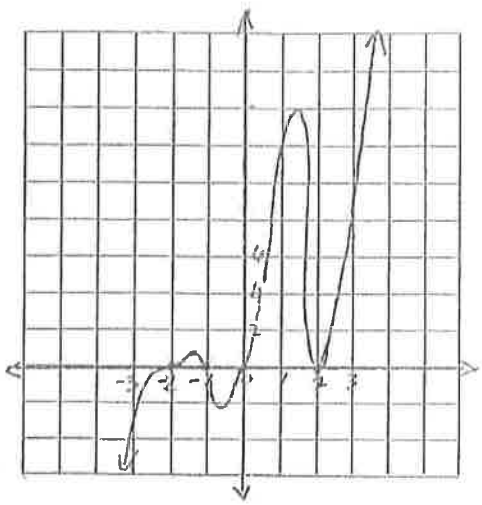
7. $3x^2 + 10x + 3$

8. $x^3 + 2x^2 - 4x - 8$

9. $x^4 - 4x^2$

10. $x^3 + 4x^2 + 4x + 1$ (hint: Use synthetic division)

POLYNOMIAL FUNCTIONS



11. For the graphed function determine the following:

a. End behavior: _____

b. List the following:

Zeros	Multiplicity	How do you know the multiplicity?

c. If the leading coefficient is ¼, write the equation of this function. Leave it in factored form.

f(x) = _____

12. For the following function, find the following features and then sketch the graph.

$f(x) = -1(x-2)(x+3)^2$

a. End behavior: _____

b. Use your calculator to find the coordinates of the points which are:

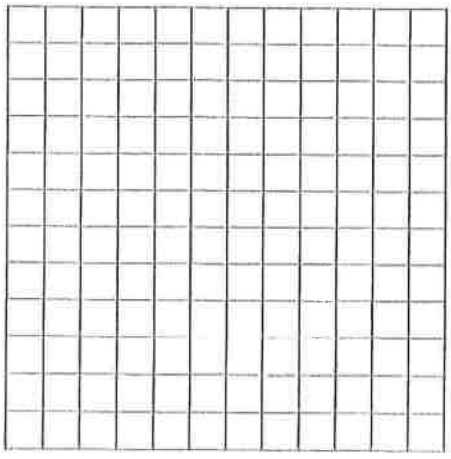
Local Maximum: _____

Local Minimum: _____

c. x-intercept(s): _____

d. State the multiplicity of your zeros from part *c* above: _____

e. y-intercept: _____



POLYNOMIAL EQUATIONS

Solve the following equations, include any imaginary solutions.

13. Solve by factoring: $2x^2 + 3x - 5 = 0$	14. Solve by using the quadratic formula: $y^2 - 3y + 4 = 0$
15. Solve by graphing: $x^4 - 10x^2 + 9 = 0$	16. By synthetic division: $x^3 + 4x^2 + 4x + 2 = 0$

17. Divide by synthetic division: $(x^3 - 4x + 2) \div (x + 2)$

18. How many solutions will the following equation have, including imaginary solutions and multiple solutions. $x^4 + 3x^3 - 6x^2 + x - 2 = 0$

19. List the possible rational roots for the following polynomial function: $y = 2x^3 - x^2 + 2x - 5$

RATIONAL FUNCTIONS

20. Match the function to the correct graph.

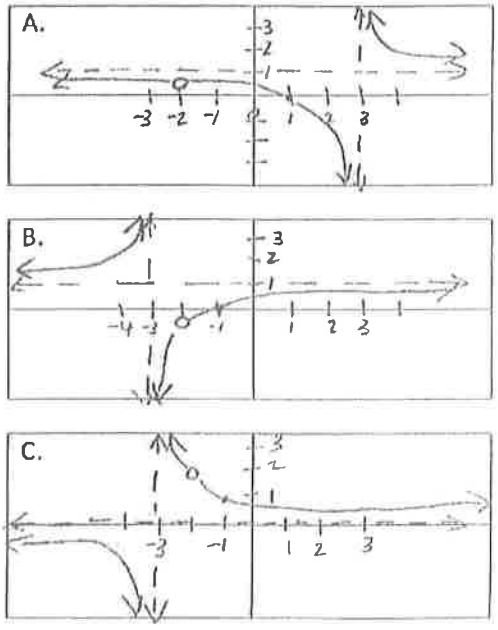
$$y = \frac{(x+2)(x-1)}{(x+2)(x+3)}$$

Describe, in words, how you can find these key without your calculator.

a. Hole?

b. Vertical asymptote?

c. Horizontal asymptote?



OPERATIONS WITH RATIONAL EXPRESSIONS

For problems 21-25, do the indicated operation, reduce if possible.

21. Reduce: $\frac{x^2+4x+4}{x^2+3x+2}$

22. Add: $\frac{x^2}{x+5}+\frac{5x}{x+5}$

23. Add: $\frac{y}{y+2}+\frac{3}{y+1}$

24: Multiply: $\frac{m^2-m}{m^2-2m+1}\cdot\frac{m^3-1}{m^2+3m}$

25: Divide: $\frac{2x+4}{x^2-4}\div\frac{x^2+2x}{3x^2-6x}$

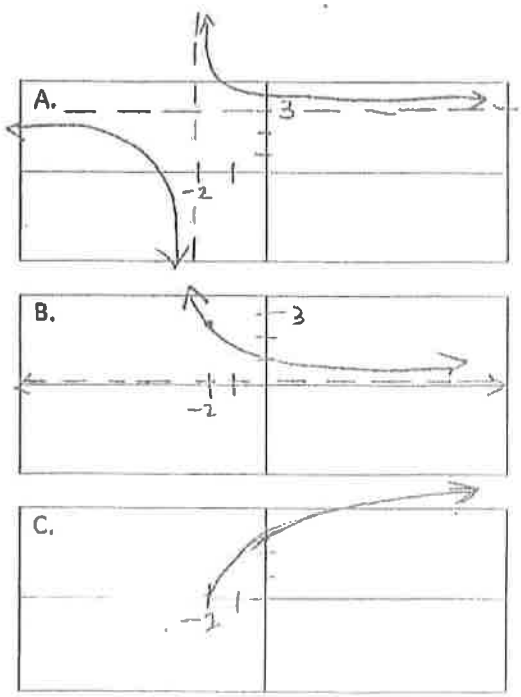
RADICAL/RATIONAL/EXPONENTIAL FUNCTIONS

Match the domain and the range to the following graphed functions.

26. _____ D: $x \in \mathcal{R}$
R: $y > 0$

27. _____ D: $x \in \mathcal{R}, x \neq -2$
R: $y \in \mathcal{R}, y \neq 3$

28. _____ D: $x \geq -2$
R: $y \geq 0$



RADICALS

29. Put in simple radical form: SQUARE ROOTS

a. $\sqrt{12}$ b. $\sqrt{8}+\sqrt{50}$ c. $\frac{1}{\sqrt{5}}$

30. Put in simple radical form: CUBE ROOTS

a. $\sqrt[3]{24}$ b. $\sqrt[3]{2x^3}+x\sqrt[3]{16}$ c. $\frac{1}{\sqrt[3]{4x}}$

31. Multiply these radicals.

a. $\sqrt{4x}\cdot\sqrt{3x}$ b. $(\sqrt{2}+\sqrt{3})^2$ c. $\sqrt[4]{9x}\cdot\sqrt[4]{9x^3}$

32. Use exponent rules to simplify.

a. $x^2 \cdot x^3$ b. $y^{1/2} \cdot y^{1/3}$ c. $(2m^{1/2} \cdot 3x) \div x^{1/2}$

33. Change to radical form: $x^{3/2}$

34. Change to exponential form: $\sqrt[5]{3x^2}$

35. Simplify. Try to not use a calculator.

a. $4^{1/2}$ b. $8^{1/3}$ c. $(\frac{1}{2})^{-2}$ d. $9^{1.5}$

36. Rational Exponent Application: When consuming an 16 ounce cup of caffeinated coffee, the percent of caffeine in the bloodstream after x hours, is given by the function $P = 100(10)^{-0.175x}$. Find the percent remaining after 5 hours.

37. Describe two ways in which you can tell if two functions are inverses of each other.

LOGARITHM and EXPONENTIAL FUNCTIONS

38. Write in logarithmic form: $3^{-2} = \frac{1}{9}$

39. Rewrite in exponential form:

a. $\log 1000 = 3$ b. $\log_2 \frac{1}{16} = -4$ c. $\ln 12 = x$

40. Expand: $\log_2 \frac{2x^2}{\sqrt[3]{y}}$

41. Rewrite as a single logarithm: $\log_2 6 + \log_2 x - 3\log_2 y - \frac{1}{2}\log_2 z$

42. Given the exponential function, $y = a \cdot b^x$, describe how you can tell if the function represents growth or decay by looking at the value, b. Use an example to show the difference between the two types of exponential function.

43. The population of a small town in Colorado is growing rapidly. City planners have recorded 12% increases per year. If the population is 1365 people this year, what will the population be in 10 years? (Use $y=a \cdot b^x$)

44. How long will it take for $100,000 to grow to $1,000,000 if the money is invested in a mutual fund earning about 9.5% per year? (Use $A = P \cdot e^{rt}$)

SOLVING EQUATIONS

Use algebra methods to solve each equation below.

45. $x^3 - x^2 - 4x + 4 = 0$

46. $\sqrt{2x+4} = \sqrt{x} + 2$

47. $(x+3)^{2/3} = 9$

48. $10{,}000 = 5{,}000\left(1 + \frac{0.06}{12}\right)^{12t}$

49. $\log_3(x-8) + \log_3 x = 2$

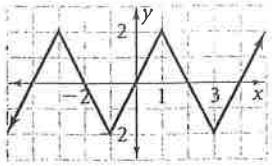
50. $\log_x 16 = -\frac{4}{3}$

Good luck on your exams!

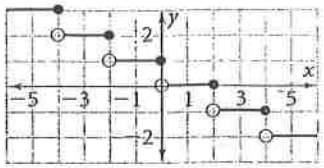
Chapter 13

Chapter Test

Determine whether each function *is* or *is not* periodic. If it is periodic, find the period and amplitude.

1. 

periodic; 4, 2

2. 

not periodic

Find the measure of an angle between 0° and 360° coterminal with the given angle.

3. $-32°$ 328°
4. $-229°$ 131°
5. $375°$ 15°

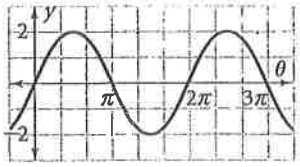
Write each measure in radians. Express the answer in terms of π and also as a decimal rounded to the nearest hundredth.

6. $-225°$
7. $120°$
8. $600°$

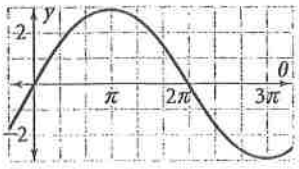
Write each measure in degrees. If necessary, round your answer to the nearest degree.

9. $\frac{5\pi}{6}$ 150°
10. -2.5π −450°
11. 0.8 46°

How many cycles does each sine function have in the interval from 0 to 2π? Find the amplitude and period of each function.

12. 

1 cycle; 2, 2π

13. 

$\frac{1}{2}$ cycle; 3, 4π

15. **Writing** Explain how to convert an angle measure in radians to an angle measure in degrees. Include an example.

16. **Physics** On each swing, a pendulum 18 inches long travels through an angle of $\frac{3\pi}{4}$ radians. How far does the tip of the pendulum travel in one swing? Round your answer to the nearest inch. 42 in.

Find the amplitude and period of each function. Then sketch one cycle of the graph of each function.

17. $y = 4 \sin (2x)$
18. $y = 2 \sin (4x)$

Solve each equation in the interval from 0 to 2π. Give an exact answer and an answer rounded to the nearest hundredth.

19. $\cos t = \frac{1}{2}$
20. $2 \sin t = \sqrt{3}$
21. $3 \tan 2t = \sqrt{3}$
22. $\cos \frac{t\pi}{4} = 1$ 0

Graph each function in the interval from 0 to 2π.

23. $y = 2 \cos x$
24. $y = -\cos \frac{\theta}{\pi}$
25. $y = 4 \sin x - 2$
26. $y = \cos (x + \pi)$
27. $y = \tan \frac{\theta}{3}$
28. $y = \tan \frac{\pi}{3}\theta$

Write an equation for each translation.

29. $y = \sin x$, 1 unit down $y = \sin x - 1$
30. $y = \cos x$, 7.5 units to the right
31. $y = \sin x$, 3 units to the left, 1.5 units down
32. $y = \cos x$, $\frac{\pi}{2}$ units to the right, 8 units up

Evaluate each expression. Write your answer in exact form. If the expression is undefined, write *undefined*.

33. $\sin 30°$ $\frac{1}{2}$
34. $\cos 60°$ $\frac{1}{2}$
35. $\sin (-330°)$ $\frac{1}{2}$
36. $\csc (-330°)$ 2
37. $\sec 270°$ undefined
38. $\tan 60°$ $\sqrt{3}$
39. $\cos 45°$ $\frac{\sqrt{2}}{2}$
40. $\cot (-60°)$ $-\frac{\sqrt{3}}{3}$

Graph each function in the interval from 0 to 2π.

41. $y = \cot \theta$
42. $y = \sec \theta + 1$
43. $y = \csc \frac{\theta}{2}$
44. $y = \csc (\theta + 1)$